Background

- Distributed data centers could be coordinated to reduce cost and emissions
- Operation faces strong uncertainties: Workload demands, ambient temperature, electricity prices, and carbon intensities

Method: Lyapunov optimization

- Prediction-free
- Can estimate the optimality gap
- Relies on careful theoretical deduction to consider operational constraints

Our contributions

- Consider emission limit and temperature management
- Develop a linear program-based method to decide the parameters of operation strategy to avoid substantial theoretical analysis

R. Xie, Y. Chen, and X. Weng, "Online Low-Carbon Workload, Energy, and Temperature Management of Distributed Data Centers," 4th Energy Conversion and Economics Annual Forum (ECE Forum 2024).

System structure



Cooling system and temperature control

$$0 \leq p_{jt}^{C} \leq P_{j}^{C}, \forall j \in \mathcal{J},$$

$$\underline{T}_{j}^{H} \leq \tau_{jt}^{H} \leq \overline{T}_{j}^{H}, \forall j \in \mathcal{J},$$

$$\tau_{j(t+1)}^{H} = \tau_{jt}^{H} + \kappa_{j}^{B} p_{jt}^{B} - \kappa_{j}^{C} p_{jt}^{C} - \beta_{jt}^{C}, \forall j \in \mathcal{J}.$$

Emission bound

$$\frac{1}{T} \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{J}} \gamma_{jt}^E p_{jt}^G \le C^E,$$

Workload flow

$$\begin{split} & 0 \leq a_{it}^{F} \leq \alpha_{it}^{F}, \forall i \in \mathcal{I}, \\ & 0 \leq m_{ijt}^{R} \leq M_{ij}^{R}, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}, \\ & 0 \leq p_{jt}^{B} \leq P_{j}^{B}, \forall j \in \mathcal{J}, \\ & q_{it}^{F} \geq 0, \forall i \in \mathcal{I}, q_{jt}^{B} \geq 0, \forall j \in \mathcal{J}, \\ & q_{i(t+1)}^{F} = q_{it}^{F} + a_{it}^{F} - \sum_{j \in \mathcal{J}} m_{ijt}^{R}, \forall i \in \mathcal{I}, \\ & q_{j(t+1)}^{B} = q_{jt}^{B} + \sum_{i \in \mathcal{I}} m_{ijt}^{R} - p_{jt}^{B}, \forall j \in \mathcal{J}. \\ & f_{t}^{W} = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \gamma_{ij}^{R} m_{ijt}^{R} + \sum_{i \in \mathcal{I}} \gamma_{i}^{F} (\alpha_{it}^{F} - a_{it}^{F}), \end{split}$$

Energy storage system

$$\begin{split} & 0 \leq p_{jt}^{SC} \leq P_{j}^{SC}, 0 \leq p_{jt}^{SD} \leq P_{j}^{SD}, \forall j \in \mathcal{J}, \\ & \underline{E}_{j}^{S} \leq e_{jt}^{S} \leq \overline{E}_{j}^{S}, \forall j \in \mathcal{J}, \\ & e_{j(t+1)}^{S} = e_{jt}^{S} + p_{jt}^{SC} \eta_{j}^{SC} - p_{jt}^{SD} / \eta_{j}^{SD}, \forall j \in \mathcal{J}. \\ & f_{t}^{S} = \sum_{j \in \mathcal{J}} \gamma_{j}^{S} (p_{jt}^{SC} + p_{jt}^{SD}). \end{split}$$

Framework of the proposed method



Performance guarantee

Theorem 1. Suppose that Assumption 1 and Assumption 2 hold. With parameter V > 0 and arbitrary θ , the optimality gap of the online algorithm is upper bounded by:

$$F^l(V,\theta) - F^m \le B/V.$$

Feasibility guarantee

Theorem 2. Suppose that Assumption 1 and Assumption 3 hold and the parameters $V \ge 0$ and θ satisfy (16). Then the strategy by Algorithm 1 is feasible in problem (10):

Parametric virtual queues

$$\begin{split} \tilde{q}_{it}^F &= q_{it}^F + \theta_i^F, \forall i \in \mathcal{I}, \\ \tilde{q}_{jt}^B &= q_{jt}^B + \theta_j^B, \tilde{q}_{jt}^S = e_{jt}^S + \theta_j^S, \tilde{q}_{jt}^H = \tau_{jt}^H + \theta_j^H, \\ \tilde{q}_{t+1}^E &= \max\left\{\tilde{q}_t^E + \sum_{j \in \mathcal{J}} \gamma_{jt}^E p_{jt}^G - C^E, 0\right\}, \end{split}$$

Parametric online algorithm

Algorithm 1: Online algorithm in time slot t.

- 1 Observe the uncertainty realizations of α_{it}^F , β_{jt}^C , γ_{jt}^G , and γ_{jt}^E .
- **2** Solve the LP problem (15) and obtain the values of $a_{it}^F, m_{ijt}^R, p_{jt}^B, p_{jt}^{SC}, p_{jt}^{SD}$, and p_{jt}^C .
- **3** Update the queues according to (13b)–(13f) and obtain $\tilde{q}_{i(t+1)}^F$, $\tilde{q}_{j(t+1)}^B$, $\tilde{q}_{j(t+1)}^S$, $\tilde{q}_{j(t+1)}^H$, and \tilde{q}_{t+1}^E .

Parameter optimization

Algorithm 2: Parameter optimization algorithm.

1 Let $Q^E = 0$.

2 Repeat:

- **3** | Solve the LP problem (17) and obtain (V, θ) .
- 4 Simulate the online algorithm using the historical dataset and let $Q^E = \max_t \tilde{q}_t^E$.
- 5 Until Q^E converges.

Case study

• Iteration process



The proposed parameter optimization algorithm converges after about 5 iterations.

• Bounds and simulated queues



The proposed method is effective in limiting emissions and ensuring feasibility.

Method comparison

- Proposed
- C1 (offline & emission bound): Impractical
- C2 (greedy & emission bound): High cost
- C3 (offline): Impractical & high emission
- C4 (greedy): High cost & high emission
- C5 (no emission bound): High emission

 Table 1: Method Comparison

Method	Cost rate $(\$/h)$	Emission rate (tCO_2/h)
Proposed	335.3	1.160
C1	251.2	1.200
C2	1378	1.200
C3	231.7	2.784
C4	457.2	2.384
C5	251.6	2.803

The proposed method has the lowest cost among the online methods that satisfy the emission bound.

Parameter sensitivity analysis





Under varying bound thresholds, the proposed method consistently satisfies emission requirements.