Predict-and-Optimize Robust Unit Commitment with Statistical Guarantees via Weight Combination

Rui Xie, Yue Chen, Pierre Pinson

R. Xie, Y. Chen, and P. Pinson, “Predict-and-optimize robust unit commitment with statistical guarantees via weight combination,” arXiv:2411.03138, 2024.

Subject: Unit commitment (UC) under renewable and demand uncertainty

Goal:

- Enhance **out-of-sample performance**
- Ensure **robustness**

Idea: Bridge the gap between **data** and **optimization**

Approach

- Data-driven robust optimization (RO) with **statistical guarantees**
  - ✓ Consider the randomness of data-driven uncertainty set construction

$$\mathbb{P}^N[\Pr[u \in \mathcal{U}] \geq 1-\varepsilon] \geq 1-\delta$$

- Integrated forecasting and optimization (**predict-and-optimize**)
  - ✓ Different from traditional prediction that minimizes the forecast error
  - ✓ Optimize the performance of the final strategy

Related Works

Optimization methods under uncertainty

- Stochastic programming (SP): Requires accurate probability distributions
- Traditional robust optimization (RO): Lacks **statistical guarantees**
- Distributionally robust optimization (DRO): Offers statistical guarantees but needs substantial data
- Data-driven RO with **dimension-free** statistical guarantee [1]: Not adapted to two-stage RO

"**Predict-and-optimize**" [2]:

- Utilized in UC [3,4]
- Lacks integration with RO and theoretical robustness guarantees

[1] L. J. Hong, Z. Huang, and H. Lam, "Learning-based robust optimization: Procedures and statistical guarantees," Management Science, 2021.
[2] A. N. Elmachtoub and P. Grigas, "Smart "predict, then optimize"," Management Science, vol. 68, no. 1, pp. 9–26, 2022.
[3] X. Chen, Y. Yang, Y. Liu, and L. Wu, "Feature-driven economic improvement for network-constrained unit commitment: A closed-loop predict-and-optimize framework," IEEE Transactions on Power Systems, 2022.
[4] H. Wu, D. Ke, L. Song, S. Liao, J. Xu, Y. Sun, and K. Fang, "A novel stochastic unit commitment characterized by closed-loop forecast-and-decision for wind integrated power systems," IEEE Transactions on Power Systems, 2024.

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- Aim: Solve a chance-constrained UC problem

Day-ahead dispatch cost

Intraday redispatch cost

$$\min_{x\in\mathcal{X},\eta} \; f(x)+\eta$$

$$\text{s.t.}\;\; \Pr\left[\min_{y\in\mathcal{Y}(x,u)} h(y)\le \eta\right]\ge 1-\varepsilon$$

- The **accurate** distribution is unknown
- Consider a **conservative approximation** **(Lemma 1)** with $\Pr[u\in\mathcal{U}]\ge 1-\varepsilon$

$$\min_{x\in\mathcal{X}} \; f(x)+\max_{u\in\mathcal{U}}\min_{y\in\mathcal{Y}(x,u)} h(y)$$

⬇ Linearized cost function and constraints

Two-stage robust UC $$\min_{x\in\mathcal{X}} \; C^T x+\max_{u\in\mathcal{U}}\min_{y:Ay\ge Bx+Du+E} F^T y$$

Data-driven uncertainty set and statistical guarantees

- What we want: An uncertainty set s.t. $\mathbb{P}^N[\Pr[u \in \mathcal{U}] \geq 1-\varepsilon] \geq 1-\delta$
- What we have: Day-ahead **prediction** $\hat{u}$ and historical **forecast error** $e_{1:N}$
- Assumption: Day-ahead forecast errors are **i.i.d. continuous** random variables
- Steps
  - ✓ Divide the historical forecast error data into two **disjoint** groups N_1 and N_2
  - ✓ N_1 determines the ellipsoid's **shape** and center
  - ✓ N_2 determines the **size** (to include enough data points in the ellipsoid)
- **Theorem 1**: The optimal solution $x_0 \coloneqq x^*_{\mathcal{U}_1}$ satisfies

$$\mathbb{P}^N\left[\Pr\left[O \leq O_{x_0} \leq O_{\mathcal{U}_1}\right] \geq 1-\varepsilon\right] \geq 1-\delta$$

Optimal value of the chance-constrained problem

Performance of the obtained solution

Optimal value of the RO problem

02 Robust UC with Statistical Guarantees

Uncertainty set reconstruction: To reduce conservativeness

- Leverage data and UC problem information
- **Lemma 1** (The **best** uncertainty set): If (x^*, η^*) is optimal in the chance-constrained problem, then $O = O_{\mathcal{U}^*}$, where

$$\mathcal{U}^* = \left\{ u \middle| \min_{y \in \mathcal{Y}(x^*, u)} h(y) \le \eta^* \right\}, \Pr[u \in \mathcal{U}^*] \ge 1 - \varepsilon$$

- Approximate $\mathcal{U}^*$ using

$$(x, \eta) = (x_0, O_{x_0} - f(x_0))$$

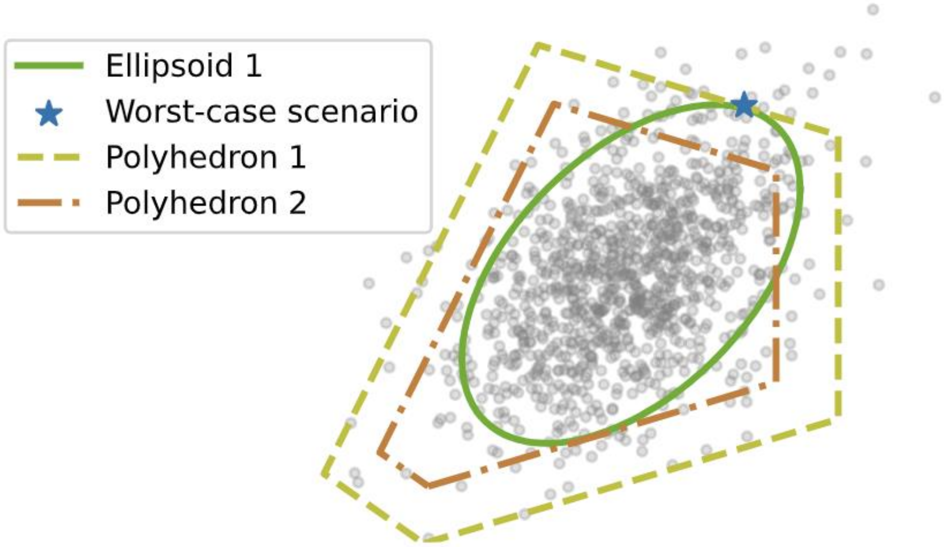
An obtained solution

Estimated performance in the historical dataset

- **Theorem 2** (Statistical guarantee):

$$\mathbb{P}^N\left[\Pr\left[O \le O_{x_1} \le O_{\mathcal{U}_2} \le f(x_0) + \beta\right] \ge 1 - \varepsilon\right] \ge 1 - \delta$$

Improve performance



Solution algorithm

- Uncertainty sets are either ellipsoidal or polyhedral
- Use the **C&CG algorithm** to solve two-stage RO problems

Algorithm 1: Solution of robust unit commitment

Input: Parameters of (9); ε; δ; $\mathcal{U}_0$; $\hat{u}$; $e_{1:N}$; N_2

Output: Unit commitment strategy x_1

Two datasets

$N_1 \leftarrow N - N_2$

Divide $e_{1:N}$ into $e^{(1)}_{1:N_1}$ and $e^{(2)}_{1:N_2}$

Construction

Calculate μ and Σ according to (13)

$\alpha \leftarrow \max\{(e_n^{(1)} - \mu)^\top \Sigma^{-1} (e_n^{(1)} - \mu) \mid n = 1, 2, \ldots, N_1\}$

$\mathcal{U}_1' \leftarrow \{u \in \mathcal{U}_0 \mid (u - \hat{u} - \mu)^\top \Sigma^{-1} (u - \hat{u} - \mu) \leq \alpha\}$

C&CG

Solve problem (9) with $\mathcal{U} = \mathcal{U}_1'$ using the C&CG algorithm and obtain the optimal solution x_0

Reconstruction

$b_n \leftarrow \min_{y: Ay \geq Bx_0 + D(\hat{u} + e_n^{(2)}) + E} F^\top y$, for $n = 1, 2, \ldots, N_2$

Arrange $b_n, n = 1, 2, \ldots, N_2$ from small to large and get $b'_n, n = 1, 2, \ldots, N_2$

$n^* \leftarrow \min\{n \mid \sum_{m=0}^{n-1} C_{N_2}^m (1 - \varepsilon)^m \varepsilon^{N_2 - m} \geq 1 - \delta\}$

$\beta \leftarrow b'_{n^*}$

$\mathcal{U}_2 \leftarrow \{u \in \mathcal{U}_0 \mid \exists y, \text{s.t. } Ay \geq Bx_0 + Du + E, F^\top y \leq \beta\}$

C&CG

Solve problem (9) with $\mathcal{U} = \mathcal{U}_2$ using the C&CG algorithm and return the optimal solution x_1

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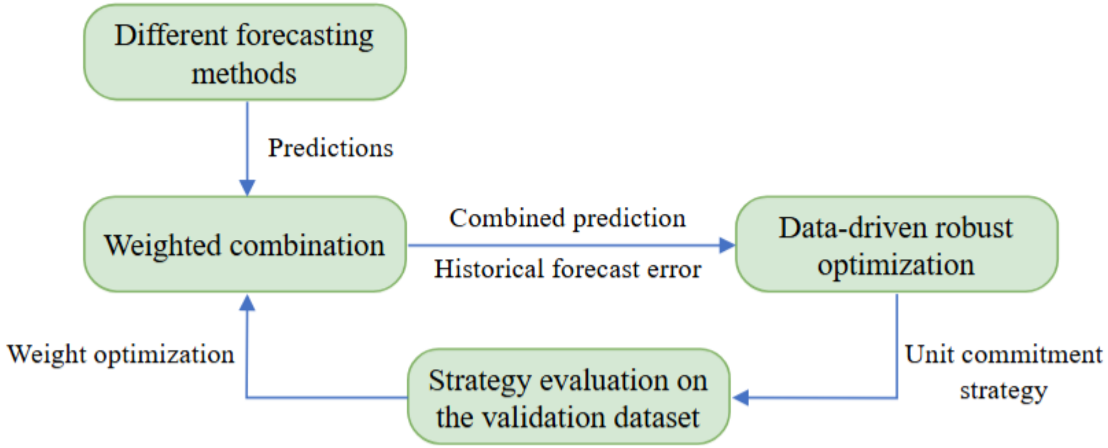
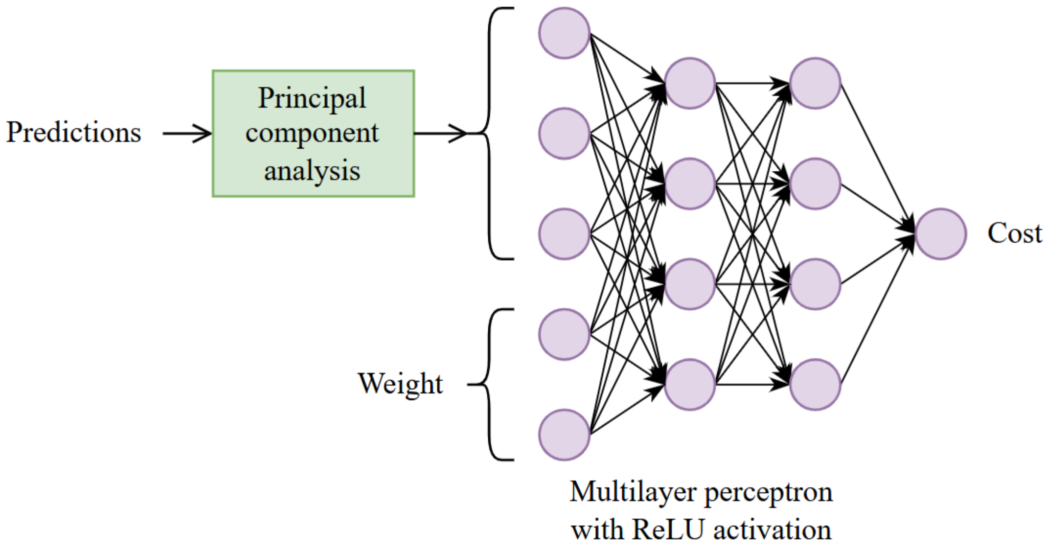
- Predict-and-optimize framework



- Construct a multilayer perceptron (MLP)-based surrogate model to **speed up** the weight optimization



MILP-based weight optimization

Case Studies: IEEE 30-Bus System

TABLE I
AVERAGE FORECAST ERRORS OF DIFFERENT METHODS

	Method	RMSE	MAE
	M1	84.39	54.64
	M2	80.93	52.37
	M3	80.44	55.23
Minimize MSE	C1	76.14	51.26
Proposed	C2 (30-bus)	76.95	52.29
	C2 (118-bus)	78.72	53.80

Prediction data

- Three forecasting methods
- Combining predictions can **enhance accuracy**

Method comparison

TABLE II
SETTINGS OF UNIT COMMITMENT METHODS FOR COMPARISON

	Method	Statistical guarantee	Integrated forecasting and optimization	Uncertainty set reconstruction
	SP	×	×	×
100%	RO1	×	×	×
95%	RO2	×	×	×
	P1	✓	✓	×
	P2	✓	×	✓
	Proposed	✓	✓	✓

TABLE III
UNIT COMMITMENT RESULTS OF DIFFERENT METHODS IN MODIFIED IEEE 30-BUS SYSTEM

Method	Objective ($)	Feasible rate	Total cost ($)	Time (s)
SP	84832	88%	82985	218
RO1	106810	100%	92652	143
RO2	97350	97%	90468	94
P1	97848	98%	89149	124
P2	90122	98%	88318	147
Proposed	89725	98%	88243	121

- SP lacks robustness
- Traditional data-driven RO does not have statistical guarantees
- The proposed method has the **lowest** objective value and test total cost among methods that have **statistical guarantees**

Case Studies: A special case of two random loads

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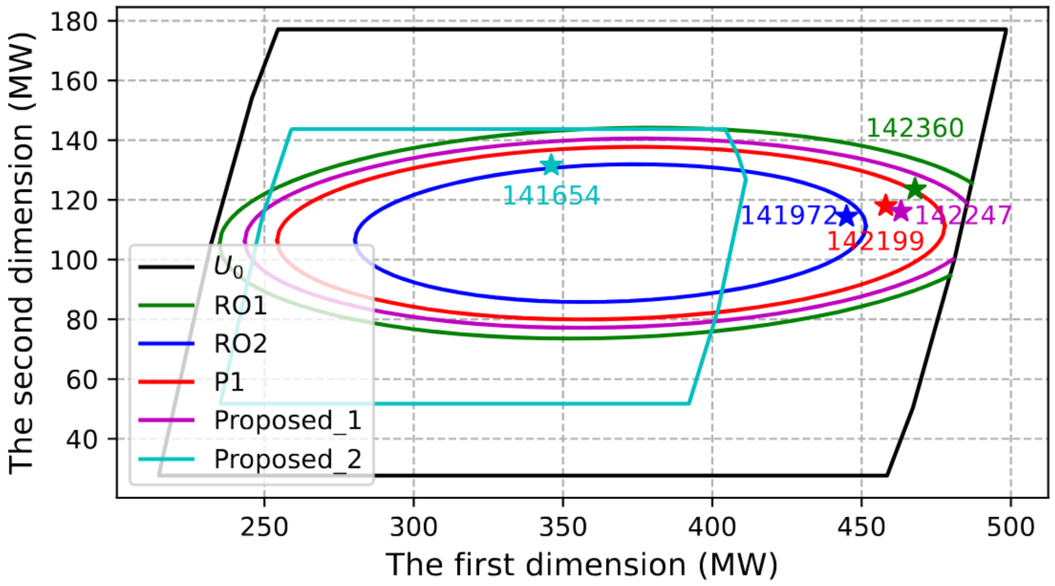
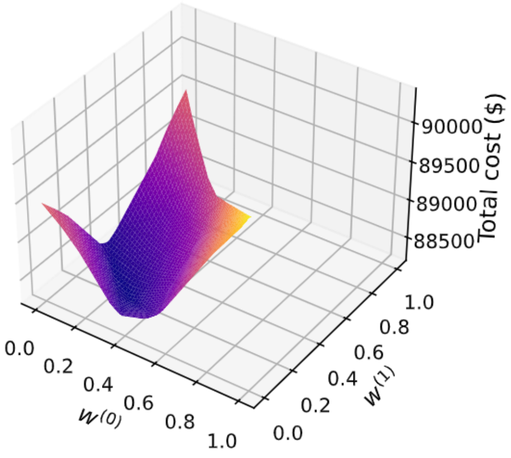
Project the uncertainty sets onto two dimensions

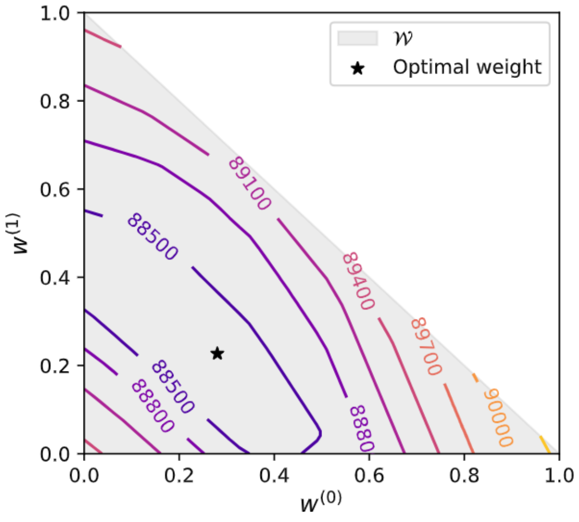
- Bound: $\mathcal{U}_0$
- RO1: 100% data points
- RO2: 95% data points
- P1: $\mathbb{P}^N[\Pr[u \in \mathcal{U}] \geq 95\%] \geq 95\%$
- **RO2 ⊂ P1 ⊂ RO1**
- Proposed_1, Proposed_2: The first and second uncertainty sets in the proposed method
- Proposed_2 excludes some **high-cost scenarios** in Proposed_1, but includes other regions to ensure the statistical guarantee



04 Case Studies: Sensitivity Analysis

Impact of weight w



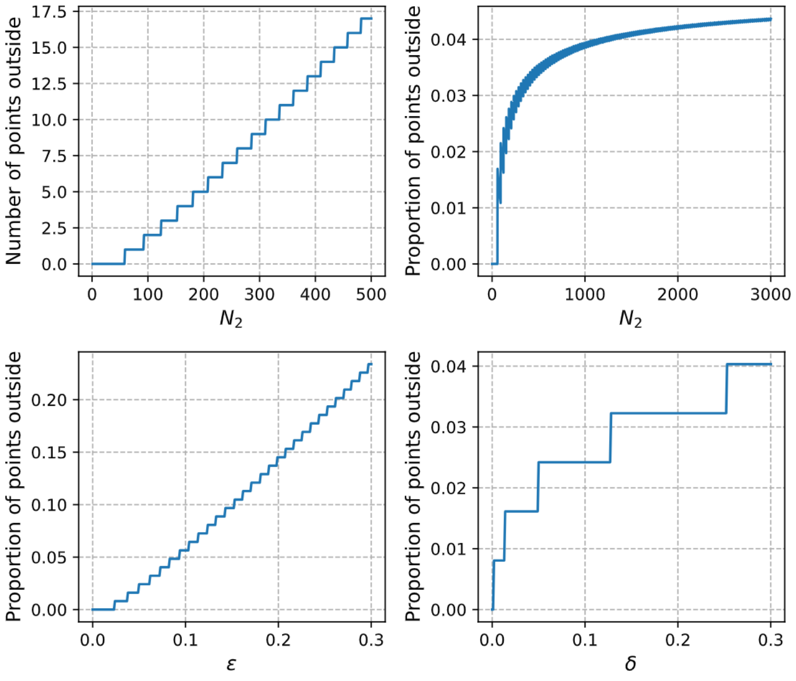


$$w^{(0)} + w^{(1)} + w^{(2)} = 1$$

$$w^{(0)}, w^{(1)}, w^{(2)} \geq 0$$

Requirements for statistical guarantee

- p: Proportion of points outside the uncertainty set
  - ✓ p ↑ ε ($N_2 \uparrow \infty$)
  - ✓ p ↑ (ε ↑) and p ≤ ε
  - ✓ p ↑ (δ ↑)



- Developed a **predict-and-optimize** two-stage robust UC method with **statistical guarantees**
  - Predict-and-optimize integration
  - Statistical guarantee

  ➡ Improve out-of-sample performance

- Case studies show that the proposed method
  - **Balances** robustness and out-of-sample performance
  - **Outperforms** traditional SP and RO methods